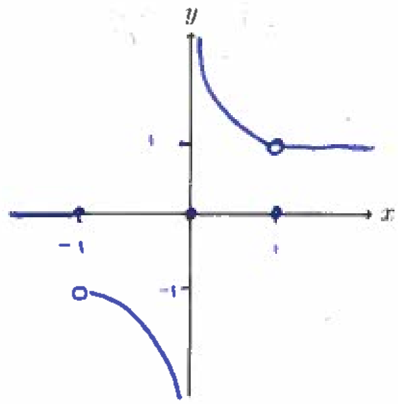
MATH 141 HOMEWORK 1 DUE DATE: THURSDAY, MAY 11TH

Name: Solutions

This homework is due Thursday, May 11th. If you have questions regarding any of this, feel free to ask during office hours. When writing solutions. present your answers clearly and neatly.

1. Graph the function

$$f(x) = \begin{cases} 0, & x \leq -1 \\ 1/x, & 0 < |x| < 1 \\ 0, & x = 0 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$$



2. Suppose the functions $f(x)$ and $g(x)$ are defined for all x and that $\lim_{x\to 0} f(x) = 1/2$ and $\lim_{x\to 0} g(x) = \sqrt{2}$. Find the limits as $x \to 0$ of the following functions.

(a) $-g(x)$

Answer: $-\sqrt{2}$

(b) $f(x) + g(x)$

Answer: $1/2 + \sqrt{2}$

(c) $x + f(x)$

Answer: $1/2$

(d) $g(x) \cdot f(x)$

Answer: $\sqrt{2}/2$

(e) $1/f(x)$

Answer: 2

(f) $\dfrac{f(x) \cdot \cos(x)}{x - 1}$

Answer: $-1/2$

3. Find the following limits.

(a) $\lim_{x\to 64} \dfrac{x^{2/3} - 16}{\sqrt{x} - 8}$ [Hint: $x - 64 = (x^{1/3} - 4)(x^{2/3} + 4x^{1/3} + 16)$]

$$\lim_{x\to 64} \frac{x^{2/3}-16}{\sqrt{x}-8} = \lim_{x\to 64} \frac{(x^{1/3}-4)(x^{1/3}+4)(x^{1/2}+8)}{x-64} = \lim_{x\to 64} \frac{(x^{1/3}-4)(x^{1/3}+4)(x^{1/2}+8)}{(x^{1/3}-4)(x^{2/3}+4x^{1/3}+16)} = \lim_{x\to 64} \frac{(x^{1/3}+4)(x^{1/2}+8)}{x^{2/3}+4x^{1/3}+16}$$

Answer: $8/3$

(b) $\lim_{x\to 0} \frac{x^2+x}{x^5+2x^4+x^3} = \lim_{x\to 0} \frac{x(x+1)}{x^3(x+1)^2} = \lim_{x\to 0} \frac{1}{x^2(x+1)}$

$\lim_{x\to 0^+} \frac{1}{x^2(x+1)} = \infty$ (since $x^2, x+1>0$)

$\lim_{x\to 0^-} \frac{1}{x^2(x+1)} = \infty$ "

Answer: ∞

(c) $\lim_{x\to -1} \frac{x^2+x}{x^5+2x^4+x^3} = \lim_{x\to -1} \frac{1}{x^2(x+1)}$

$\lim_{x\to -1^+} \frac{1}{x^2(x+1)} = \infty$ (since $x^2, x+1>0$) $\lim_{x\to -1^-} \frac{1}{x^2(x+1)} = -\infty$ (since $x^2>0$, $x+1<0$).

Answer: DNE

4. Find the following limits, as $x \to \infty$.

(a) $\lim_{x\to\infty} \frac{2x^2+3}{5x^2+7}$ $2x^2+3 = f(x)$, $5x^2+7 = g(x)$

deg $f(x)$ = deg $g(x)$

Answer: $2/5$

(b) $\lim_{x\to\infty} \frac{x^4+x^3}{12x^3+128}$ $x^4+x^3 = f(x)$, $12x^3+128 = g(x)$

$f(x) > 0$ as $x \to \infty$
$g(x) > 0$ as $x \to \infty$

deg $f(x)$ > deg $g(x)$

Answer: ∞.

5. **Lorentz contraction** In relativity theory, the length of an object, say a rocket, appears to an observer to depend on the speed at which the object is traveling with respect to the observer. If the observer measures the rocket's length as L_0 at rest, then at speed v the length will appear to be

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

This equation is the Lorentz contraction formula. Here, c is the speed of light in a vacuum.

(a) What happens to L as v increases?

Answer: Decreases

(b) Find $\lim_{v\to c^-} L$.

$\lim_{v\to c^-} L_0\sqrt{1-v^2/c^2} = L_0\sqrt{1-\lim_{v\to c} v^2/c^2} = L_0\sqrt{1-1} = 0$

Answer: 0.

The End.